Exam. Code : 103204
Subject Code : 1140

B.A./B.Sc. 4th Semester

MATHEMATICS

Paper—II

(Solid Geometry)

Time Allowed—Three Hours] [Maximum Marks—50

Note :— Attempt any **FIVE** questions, selecting at least **TWO** questions from each section.

SECTION—A

I. (a) The plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the co-ordinate axes in A, B, C. Prove that the equation to the cone generated by the line drawn from O to meet the circle ABC is

$$yz\left(\frac{b}{c}+\frac{c}{b}\right)+zx\left(\frac{c}{a}+\frac{a}{c}\right)+xy\left(\frac{a}{b}+\frac{b}{a}\right)=0.$$ 5

(b) Find the equation of the right circular cone generated when the straight line $2y + 3z = 6$, $x = 0$ revolves about z-axis. 5

II. (a) Find the equation of the cone circumscribing the sphere $x^2 + y^2 + z^2 + 2x - 2y - 2 = 0$ and having its vertex at (1, 1, 1). 5

(b) Prove that the equation $\sqrt{fx}+\sqrt{gy}+\sqrt{hz}=0$ represents a cone which touches the co-ordinate planes and that the equation of reciprocal cone is $fyz + gzx + hxy = 0$. 5

III. (a) Find the value of λ if the plane $\lambda x + y + z = 0$ cuts the cone $xy + yz + zx = 0$ in perpendicular lines. 5

(b) Find the equation of the cone passing through the coordinate axes and the three mutually perpendicular lines :

$$\frac{1}{2}x=y=-z,\ x=\frac{1}{3}y=\frac{1}{5}z \text{ and } \frac{1}{8}x=-\frac{1}{11}y=\frac{1}{5}z.$$

5

IV. (a) Find the equation to the cylinder whose generators are parallel to the line $\frac{x}{\ell}=\frac{y}{m}=\frac{z}{n}$ and base the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, $z = 0$. 5

(b) Find the equation of the right circular cylinder of radius 2 whose axis is the line $\frac{x-1}{2}=y-2=\frac{z-3}{2}$. 5

V. (a) A cylinder cuts the plane $z = 0$ in the curve $x^2+\frac{y^2}{4}=\frac{1}{4}$, and has its axis parallel to $3x = -6z$. Find its equation. 5

(b) Show that the angle between the lines $x + y + z = 0$, $ayz + bzx + cxy = 0$ is $\frac{\pi}{2}$ if $a + b + c = 0$. 5

SECTION—B

VI. Identify the surface represented by $4x^2 + 9y^2 + 16z^2 = 144$. Trace it roughly. Also find the area of plane curve in which $y = 2$ cuts it. 10

VII. Prove that :

$5x^2 - 16y^2 + 5z^2 + 8yz - 14zx + 8xy + 4x + 20y + 4z - 24 = 0$

represents hyperbolic paraboloid. 10

VIII. (a) Show that the locus of the foot of perpendicular from the centre of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ to any of its tangent planes is $(x^2 + y^2 + z^2)^2 = a^2x^2 + b^2y^2 + c^2z^2$. 5

(b) Prove that the feet of the six normals from (α, β, γ) to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ lie on the cone

$$\frac{a^2(b^2-c^2)}{x}\alpha+\frac{b^2(c^2-a^2)}{y}\beta+\frac{c^2(a^2-b^2)}{z}\gamma=1.$$

5

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IX. (a) Find the locus of points from which three mutually perpendicular tangent lines can be drawn to the conicoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1.$ 5

(b) Show that the plane $ax + by + cz + d = 0$ touches the surface $px^2 + qy^2 + 2z = 0$ if $\frac{a^2}{p}+\frac{b^2}{q}+2cd=0$. 5

X. Show that the feet of the normals from the point (α, β, γ) to paraboloid $x^2 + y^2 = 2az$ lie on the sphere $x^2 + y^2 + z^2 - z(\alpha + \gamma) - \frac{y}{2\beta}(\alpha^2 + \beta^2) = 0$. 10